2m/MTH–150 Syllabus–2023

2025

(May–June)

FYUP : 2nd Semester Examination

MATHEMATICS

(Minor)

(Fundamental Mathematics—II)

(MTH–150)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* If by rotation of the rectangular axes the equation $17x^2 + 18xy - 7y^2 = 1$ reduces to the form $ax^2 + by^2 = 1$, then find the values of a and b. Also, find the angle through which the axes are rotated. 4+1=5

(b) Find the equation of the pair of straight lines through the point (2, –3) parallel to the straight lines

$$15x^2 + xy - 6y^2 + x + 7y - 2 = 0$$ 4

(c) Reduce the equation

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

to standard form. 5

(d) Show that the angle between one of the lines $ax^2 + 2hxy + by^2 = 0$ and one of the lines $(a-\lambda)x^2 + 2hxy + (b-\lambda)y^2 = 0$ is equal to the angle between the other two lines of the system. 4

2. (a) Find the angle through which a set of rectangular axes must be turned, without the change of origin, so that the expression $7x^2 + 4xy + 3y^2$ will be transformed into the form $a'x^2 + b'y^2$. 5

(b) Find the value of k so that the equation

$$kx^2 + 3xy - 5y^2 + 7x + 14y + 3 = 0$$

may represent a pair of straight lines. 5

(c) Find the condition that the pair of lines

$$Ax^2 + 2Hxy + By^2 = 0$$

may be conjugate diameters of the conic

$$ax^2 + 2hxy + by^2 = 1$$ 4

(d) Show that the equation

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

represents an ellipse and find its centre. 1+3=4

UNIT—II

3. (a) Find the equation of the plane passing through the point (1, –2, 3) and perpendicular to the lines

$$\frac{x-1}{4} = \frac{y+2}{5} = \frac{z}{2} \text{ and } \frac{x+4}{3} = \frac{y}{-2} = \frac{z-7}{1}$$ 4

(b) Find the equation of the sphere passing through the origin and the circle

$$x^2 + y^2 + z^2 - 4x + 3 = 0 = x^2 + y^2 + z^2 + 2y + 4$$ 5

(c) Show that the cones $ax^2 + by^2 + cz^2 = 0$ and $bcx^2 + cay^2 + abz^2 = 0$ are reciprocal to each other. 5

(d) Find the equation of the cone whose vertex is the point (1, 1, 1) and whose guiding curve is $z = 0$, $x^2 + y^2 = 4$. 5

4. (a) Find the normal form of the plane which passes through the point (–1, 5, 2) and whose normal has direction ratios 3, 0, 4. 5

(b) Find the enveloping cone of the sphere

$$x^2+y^2+z^2+2x-4y+2z+1=0$$

with vertex at (2, 1, 3). 5

(c) Prove that the plane $2x-2y+z+12=0$ touches the sphere

$$x^2+y^2+z^2-2x-4y+2z=3$$ 4

(d) Find the equation of a right circular cone whose vertex is $(1, -2, -1)$, semi-vertical angle is $\frac{\pi}{3}$ and axis is

$$\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z+1}{5}$$ 5

UNIT—III

5. (a) Show that

$$\lim_{(x,\,y)\to(0,\,0)} \frac{xy}{\sqrt{x^2+y^2}}=0$$ 4

(b) Determine whether the function $f:\mathbb{R}^2\to\mathbb{R}$ given by

$$f(x,\,y)=xy\ln(x^2+y^2)$$

has a removable discontinuity at (0, 0). 5

(c) Determine whether

$$\lim_{(x,\,y)\to(0,\,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2}$$

exists or not. 5

(d) Show that

$$\frac{\partial^2 u}{\partial x\,\partial y}=\frac{\partial^2 u}{\partial y\,\partial x}$$

when $u=x^y$. 5

6. (a) (i) Let $f:\mathbb{R}^2\to\mathbb{R}$ be given by

$$f(x,\,y)=\frac{xy}{x^2+y^2}$$

when $(x,\,y)\neq(0,\,0)$ and $f(0,\,0)=0$. Find the partial derivatives $f_x(0,\,0)$ and $f_y(0,\,0)$.

(ii) Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by $f(x,\,y)=x^2+y^2+xy$. Show that

$$f_{xy}=f_{yx}$$ 5+4=9

(b) Determine the second-order partial derivatives $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x\,\partial y}$ for

$$f(x,\,y)=x^2e^y.$$ 5

(c) If $z=\frac{y}{x+y}$, then find the value of

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}.$$ 5

UNIT—IV

7. (a) Prove that

$$\bar{a}\times\{\bar{b}\times(\bar{c}\times\bar{d})\}=(\bar{b}\cdot\bar{d})\bar{a}\times\bar{c}-(\bar{b}\cdot\bar{c})\bar{a}\times\bar{d}$$ 4

(b) Prove that any vector $\bar{r}$ can be expressed as $\bar{r}=\frac{[\bar{r}\,\bar{b}\,\bar{c}]}{[\bar{a}\,\bar{b}\,\bar{c}]}\bar{a}+\frac{[\bar{r}\,\bar{c}\,\bar{a}]}{[\bar{a}\,\bar{b}\,\bar{c}]}\bar{b}+\frac{[\bar{r}\,\bar{a}\,\bar{b}]}{[\bar{a}\,\bar{b}\,\bar{c}]}\bar{c}$, where $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors. 5

(c) Prove that

$$\frac{d}{dt}\left(\bar{r}\times\frac{d\bar{r}}{dt}\right)=\bar{r}\times\frac{d^2\bar{r}}{dt^2}$$ 2

(d) If $\bar{r}=\bar{a}e^{nt}+\bar{b}e^{-nt}$, where $\bar{a}$, $\bar{b}$ are constant vectors, then show that

$$\frac{d^2\bar{r}}{dt^2}-n^2\bar{r}=0$$ 3

(e) Find the unit tangent vector $\overline{T}(t)$ and unit normal vector $\overline{N}(t)$ for the circular helix $x=a\cos t$, $y=a\sin t$, $z=ct$. 2+3=5

8. (a) Find the unit vector normal to the surface $x^2+y^2-z=0$ at the point $(-2, 1, 0)$. 3

(b) What is the greatest rate of increase of $u=xyz^2$ at the point $(1, 0, 3)$? 4

(c) Prove that

$$\nabla^2\left(\frac{1}{r}\right)=0$$ 4

(d) If $\overline{F}=(x+y+1)\hat{i}+\hat{j}+(-x-y)\hat{k}$, then prove that $\overline{F}\cdot\text{curl}\,\overline{F}=0$. 5

(e) If A and B are irrotational, then prove that $\overline{A}\times\overline{B}$ is solenoidal. 3

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